Abstract

The objective of the work is to study some problems in EDP that involve non-local operators. It is divided into two parts. In part 1, we establish the existence results of a one-dimensional periodic solution to equations with the fractional Laplacian of order $s \in (1/2, 1)$, singular nonlinearity, and gradient term under various situations, including nonlocal contra-part of classical Lienard vector equations, as well other nonlocal versions of classical results know only in the context of second-order ODE. Our proofs are based on degree theory and Perron's method. Before that we need to establish a variety of priori estimates under different assumptions on the nonlinearities appearing in the equations. Besides, we obtain also multiplicity results in a regime where a priori bounds are lost and bifurcation from infinity occurs. In the second part, we establish Ambrosetti–Prodi type results for periodic solutions of one-dimensional nonlinear problems with drift term and driftless whose principal operator is the fractional Laplacian of order $s \in (0, 1)$. We establish conditions for the existence and nonexistence of solutions. The proofs of the existence results are based on the sub-supersolution method combined with topological degree type arguments. We also establish a priori bounds in order to get multiplicity results. We also prove that the solutions are $C^{1,\alpha}$ under some regularity assumptions in the nonlinearities, that is, the solutions of equations are classical. We finish the work obtaining existence results for problems with the fractional Laplacian with singular nonlinearity. In particular, we establish an Ambrosetti-Prodi type problem with singular nonlinearities.