Composite Monotone Inclusions in Vector Subspaces: Theory, Splitting, and Applications

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Abstract

In this thesis we aim at solving primal-dual coupled inclusions in real Hilbert spaces involving the sum of different types of monotone operators. Our objective is to propose globally convergent splitting algorithms for solving the coupled inclusions, which take advantage of the intrinsic properties of each operator in the inclusion. We split this thesis in two main parts.

In the first part, we propose a splitting method for solving primal-dual inclusions involving general maximally monotone operators and linear compositions. First, we provide a generalization of the Douglas-Rachford splitting and the primal-dual algorithm, including critical step-sizes. We also derive a new Split-ADMM by applying our method to the dual of a convex optimization problem. Next, we derive an extension on Krasnosel'skiĭ-Mann (KM) iterations defined in the range of linear operators. We prove that the relaxed primal-dual algorithm with critical step-sizes defines KM iterations in the range of a particular linear operator and we derive its convergence. At the end of the first part, we provide the resolvent computation of the parallel composition of a maximally monotone operator by a linear operator under mild assumptions. This operation naturally appears when dealing with linearly composed maximally monotone operators. Additionally, in the context of convex optimization, we obtain the proximity operator of the infimal postcomposition under mild qualification conditions.

In the second part, we propose splitting methods for solving inclusions involving the sum of cocoercive, monotone-Lipschitzian, monotone continuous operators, and the normal cone to a closed vector subspace of a real Hilbert space. First, we suppose that the monotone continuous operator is zero and we provide a method generalizing the method of partial inverses and the forward-backward-half forward splitting, among others. Finally, we provide a splitting method for solving the case when the monotone continuous operator is not zero but the vector subspace is the whole Hilbert space. We obtain a generalization of the forward-back-half forward and Tseng's splitting algorithms involving line search. Also, we derive a method for solving non-linearly constrained composite convex optimization problem.

Additionally, in each section we provide numerical simulations to compare our methods with best competitors in literature. We provide simulations in total variation image restoration, sparse minimization, and constrained total variation least-square problems.