Abstract

Mathematical modeling has a key role in the description of large part of phenomena in applied science, technological and industrial applications.

A mathematical model is a set of mathematical relations, usually equations, able to describe the essential features of a natural or artificial system, with the purpose to describe, forecast and control its evolution.

The goal of this thesis is to study some control problems in mathematical models governed by partial differential equations of parabolic type. In Chapter one we introduce in a general way the problems that have been studied and the obtained main results.

In Chapter two the Single Particle Model is used to describe the behavior of a Li-ion battery. The main goal is to design a feedback input current in order to regulate the State of Charge (SOC) to a prescribed reference trajectory. In order to do that, we use the boundary ion concentration as output. First, we measure it directly and then we assume the existence of an appropriate estimator, which has been established in the literature using voltage measurements. By applying backstepping method and Lyapunov tools, we are able to build observers and to design output feedback controllers giving a positive answer to the SOC tracking problem. We provide convergence proofs and perform some numerical simulations to illustrate our theoretical results.

The Chapter three is devoted to study the boundary controllability property of some parabolic-elliptic systems. More precisely, along this chapter, we prove the null controllability property for two one-dimensional parabolic-elliptic systems. Both of them under the action of one scalar control at the boundary. In a first case, we study the null controllability for a system with a non-linear term in the parabolic part with a control placed at the boundary of the parabolic equation. In a second case, we study a linear system with the control placed at the boundary of the elliptic part. The arguments, in the first case, rely on the controllability-observability duality principle and a suitable Carleman estimate for the solution of the adjoint equation of the linearized system. Then, by means of a local inverse theorem we prove the result for the original system. For the second case, we use the moment method and the spectral analysis of the underlying spatial operator associated to such system.

In Chapter four, we address the problem of rapid stabilization of an one dimensional unstable heat equation under the action of an unknown boundary disturbance. Combining the backstepping method and the multivalued operator $sign(\cdot)$, we design a boundary feedback law which exponentially stabilizes the system, in the L^2 norm. Moreover, the

rate can be fixed arbitrarily large. The existence of solutions to the closed-loop system is obtained by using the theory of maximal monotone operators and numerical simulations are performed in order to illustrate our results.

Finally, in Chapter five, we collect some conclusion and remarks on every problem studied in the previous chapters. Besides, we discuss some remaining open questions and future line research, for every one of those problems.

Keywords: Control systems, Partial Differential Equations, Output Tracking, Backstepping Method, Controllability, Carleman estimate, Maximal Monotone Operators.