

PRESENTATION AND SUMMARY

This document collects some of the original research produced as part of my work during the last three years. As such, it aims to fulfill the requirements to obtain the degree of Doctor en Matemáticas from the Consorcio UV-PUCV-USM.

The problems and the theory presented here can be regarded as a contribution to the understanding of Markov processes that evolve in changing state-spaces. More generally, the models discussed and analyzed in the next chapters illustrate the kind of techniques needed to take account of some natural *enlargements* of the state-space of otherwise firmly settled Markovian models. Random walks on graphs, multidimensional diffusions, Lévy-driven ecological models: these is the kind of well-understood Markov processes that, in the next chapters, we will see *embedded* in larger, evolving landscapes.

For the sake of exposition, the presentation of the main results is split into chapters, more or less in accordance with the chronological production of papers I've written during the last three years. Chapter 1 presents some standard (and less standard) results on the theory of Markov process that will be used throughout the subsequent chapter. This corpus of results pretends to address a double goal: first, to serve as a quick reference for the main theorems invoked in the other chapters, and secondly to settle the basis of a pedagogical toolkit for future usage. Furthermore, this first chapter provides a survey of the main original results found thereafter. In Chapter 2 we consider our first model on a growing domain, namely simple random walk on a growing sequence of graphs, and on this setting we study some graph-covering properties. In Chapter 3, we develop a general analytic framework to allow a rigorous treatment of diffusions that evolve on changing Euclidean dimension, and presents an application to a model of trait evolution in families of species with mutation and gene migration. Chapter 4 presents an extension of the well-known multi-species stochastic Lotka-Volterra model: we introduce Lévy-type noise into the parameters of the equations, and study conditions to ensure the convergence of the abundance process to its unique stationary distribution. This model serves as basis for Chapter 5, which presents some of the ongoing efforts to embed the previously described ecological model in a setting where the very interaction strengths between the species change over time. We also present some preliminary applications of this dynamic food-web model to the study of the phenomenon known as prey-switching in the ecological literature.