Summary

The Langlands program plays an important role in Number theory and Representation Theory. A crucial aspect of this program is the functoriality conjecture, expressed in a letter of Langlands to Weil in 1967. Let F be a global field with ring of adèles $\mathbb{A}_F$ and let

$$\rho\colon {}^LG \to {}^LH,$$

be a given L-homomorphism between the L-groups of two connected (quasi-split) reductive groups **G** and **H** over F. Then, according to this conjecture, for every cuspidal automorphic representation $\pi = \bigotimes_x \pi_x$ of $\mathbf{G}(\mathbb{A}_F)$, there exists an automorphic representation $\Pi = \bigotimes_x \Pi_x$ of $\mathbf{H}(\mathbb{A}_F)$ such that, at almost all places x where π_x is unramified, Π_x is unramified and its Satake parameter corresponds to the image under ρ of the Satake parameter of π_x. Such representation will be called a weak lift or transfer of π. Furthermore that transfer process should respect arithmetic information coming from γ-factors, L-functions and ε-factors, and leads to a local version of functoriality at the ramified places as well.

When **G** is a classical group, ${}^LG$ has a natural representation into ${}^LH$ for a specific general linear group **H**, and that case has been studied by many people. When F is a number field, two main tools have been used: converse theorem and trace formulas. The former was used by Cogdell, Kim, Piateski-Shapiro and Shahidi in combination with the Langlands-Shahidi method to prove the conjecture for a globally generic automorphic representation π when **G** is a quasi-split symplectic, unitary or special orthogonal group. For the latter, Arthur and his continuators used trace formulas to get more complete results, not restricted to quasi-split groups in characteristic zero.

Lomelí extended the converse theorem method to global function fields, getting functoriality for globally generic automorphic representations of split classical groups and unitary groups. The present thesis further extends the converse theorem method, over a function field F, to establish the functoriality conjecture when **G** is a quasi-split non-split even special orthogonal group, and π a globally generic representation.

Theorem. *Let F be a global function field and π be a globally generic cuspidal automorphic representation of $\mathbf{SO}_{2n}^*(\mathbb{A}_F)$. Then, π transfers to an irreducible automorphic*

representation Π *of* $\mathbf{GL}_{2n}(\mathbb{A}_F)$. *Furthermore,* Π *can be expressed as an isobaric sum*

$$\Pi = \Pi_1 \boxplus \cdots \boxplus \Pi_d,$$

where each Π_i *is a unitary self-dual cuspidal automorphic representation of* $\mathbf{GL}_{N_i}(\mathbb{A}_F)$ *for some* N_i*, and where* $\Pi_i \not\cong \Pi_j$ *for* $i \neq j$. *Moreover if we write* $\Pi = \bigotimes_x \Pi_x$*, then for* τ_x *an irreducible generic unitary representation of* $\mathbf{GL}_m(F_x)$

$$\gamma(s, \pi_x \times \tau_x, \psi_x) = \gamma(s, \Pi_x \times \tau_x, \psi_x)$$
$$L(s, \pi_x \times \tau_x) = L(s, \Pi_x \times \tau_x)$$
$$\varepsilon(s, \pi_x \times \tau_x, \psi_x) = \varepsilon(s, \Pi_x \times \tau_x, \psi_x),$$

where L*-functions,* γ*-factors and* ε*-factors on the right are obtained by the Rankin-Selberg method and those on the left by the Langlands-Sahidi method, as extended by Lomelí to positive characteristic.*

As in Cogdell, Kim, Piateski-Shapiro and Shahidi and Lomelí, the method of proof uses converse theorem and L-functions to construct an automorphic representation of $\mathbf{GL}_n(\mathbb{A}_F)$: we provide a proof of a twisted version in positive characteristic of the converse theorem of Cogdell and Piateski-Shapiro. To apply the converse theorem, one needs analytic properties of the Langlands-Shahidi L-functions, and to establish them we adapt Lomelí's arguments to our new case. We first obtain a weak lift which has the desired properties at almost all places. Then further properties of partial L-functions give that there is a lift which is an isobaric sum of unitary cuspidal automorphic representations. It is a strong lift, since we prove the compatibility between the local factors of π and the lift Π at all places.

As an application of the functoriality and the validity of the Ramanujan conjecture for general linear groups established by L. Lafforgue, we prove the Ramanujan conjecture for globally generic cuspidal automorphic representations of our classical group in positive characteristic.

Theorem. *Let* $\pi = \bigotimes_x \pi_x$ *be a globally generic cuspidal automorphic representation of* $\mathbf{SO}^*_{2n}(\mathbb{A}_F)$. *Then every* π_x *is tempered. In particular, if* π_x *is unramified, its Satake parameters have absolute value 1.*