

# Abstract

The purpose of this thesis is the study of control properties for infinite-dimensional systems described by partial differential equations. Through the results obtained in this work, we wanted to make a contribution to the control theory of systems of partial differential equations of dispersive type putting special attention to the number of controls used in each case.

In Chapter 2 we study the exact controllability of a system composed by  $N$  Korteweg-de Vries equations. This model is known in the literature as the KdV equation on a finite star-shaped network which is used to model for instance the cardiovascular system. The system originally was controlled in the literature considering  $N$  Korteweg-de Vries equations and  $N + 1$  controls :  $N$  controls at the ends of the network and one control in the center of the network. Our first result is a response to the open problem of controlling this system with fewer controls. We prove that the system remains controllable without the control acting in the center of the network. Thus, we prove the exact controllability of the system with  $N$  controls.

The Chapter 3 is devoted to the study of the controllability of a system that couples three Korteweg-de Vries equations posed on a finite interval. The system studied here is called in the literature the generalized Hirota-Satsuma system. We prove the well-posedness of the system with three boundary control inputs. The exact controllability results for both linear and nonlinear are obtained using the exact buildability of a single Korteweg-de Vries equation and a fixed point argument.

In Chapter 4 we study a parabolic system coupling two fourth-order equations. We prove the well-posedness of the system with some regularity results. Then, we study the null controllability with two controls, one control on each equation. In this chapter, we present a result underway of the use of the algebraic solvability method to remove the control on the last equation. To obtain the null controllability with only one internal control distributed in a nonempty open subset of the domain, we require a Carleman estimate with non-homogeneous data that is not yet proven.

In Chapter 5 we study the inverse problem of retrieving the anti-diffusion coefficient

of a system formed by two Kuramoto-Sivashinsky type equation from the measurements of the solution on a part of the boundary and also at some positive time in the whole space domain. We prove the local stability of the inverse problem by using the Bukhgeim-Klibanov method and a global Carleman estimate.

**Keywords :** Exact controllability, Null controllability, Fictitious control method, Korteweg-de Vries equation, Kuramoto-Sivashinky equation, fourth-order parabolic system, Carleman estimates, Algebraic solvability.