Abstract

The purpose of this thesis is to contribute to the control of high–order partial differential equations. Each of its chapters correspond to a different contribution.

In Chapter 2 we study the null controllability of the linear Kuramoto–Sivashinsky equation by means of either boundary or internal controls. In the Dirichlet case, we apply the moment theory to prove that the null controllability holds with a single boundary control if and only if the anti–diffusion parameter of the equation does not belong to a critical set of parameters. Regarding the Neumann case, we prove that the null controllability does not hold with a single boundary control. Nevertheless, it does always hold when either two boundary controls or an internal control is considered. The proof of the latter is based on the controllability–observability duality and a Carleman estimate.

In Chapter 3 we study the cost of null controllability of a fourth–order parabolic equation. Our interest is to know the behavior of the cost in terms of the diffusion coefficient of the equation, which is denoted by $\varepsilon \in \mathbb{R}^+$. When the control time is large, we prove that the cost decreases exponentially to zero as $\varepsilon \to 0^+$. When the control time is small, on the contrary, we prove that the cost increases exponentially to infinity as $\varepsilon \to 0^+$.

In Chapter 4 we prove the local exact controllability to the trajectories of the Cahn–Hilliard equation, which is a nonlinear fourth–order parabolic equation, by means of a control supported on an interior open interval. To prove this result we derive a Carleman estimate that allows us to conclude, thanks to a duality argument, the null controllability of the linearized equation around a given solution. Then, we apply a local inversion theorem to extend the control result to the nonlinear equation.

In Chapter 5 we address the problem of null controllability of a damped wave equation, posed on the N–dimensional torus $\mathbb{T}^N = \mathbb{R}^N/\mathbb{Z}^N$, $N \in \mathbb{N}$, by means of a control supported on a suitable region of $\mathbb{T}^N$. To facilitate our analysis we decompose the damped wave equation into a system coupling a parabolic equation with an ordinary differential equation (ODE). The characteristics of the ODE do not propagate in the space directions, thus obstructing the null controllability of the coupled system when the support of the control is fixed. Under the condition that the support of the control moves and visits the whole $\mathbb{T}^N$, thus ensuring that the characteristics of the ODE reach the support of the control, we derive a Carleman estimate for the adjoint system associated to the coupled system, thus allowing us to deduce its null controllability thanks to the controllability–observability duality.

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Keywords. Kuramoto–Sivashinky equation, fourth–order parabolic equation, Cahn–Hilliard equation, damped wave equation, coupled parabolic–transport system, null controllability, boundary control, internal control, moving control, moment theory, Carleman estimates.