Abstract.

In a first part, we focus on gradient dynamical systems governed by non-smooth but also non-convex functions, satisfying the so-called Kurdyka-Łojasiewicz inequality. After obtaining preliminary results for a continuous steepest descent dynamic, we study a general descent algorithm. We prove, under a compactness assumption, that any sequence generated by this general scheme converges to a critical point of the function. We also obtain new convergence rates both for the values and the iterates. The analysis covers alternating versions of the forward-backward method, with variable metric and relative errors. As an example, a non-smooth and non-convex version of the Levenberg-Marquardt algorithm is detailed. Applications to non-convex feasibility problems, and to sparse inverse problems are discussed.

In a second part, the thesis explores descent dynamics associated to constrained vector optimization problems. For this, we adapt the classic steepest descent dynamic to functions with values in a vector space ordered by a solid closed convex cone. It can be seen as the continuous analogue of various descent algorithms developed in the last years. We have a particular interest for multi-objective decision problems, for which the dynamic make decrease all the objective functions along time. We prove the existence of trajectories for this continuous dynamic, and show their convergence to weak efficient points. Then, we explore an inertial dynamic for multi-objective problems, with the aim to provide fast methods converging to Pareto points.